A Novel Multi-Attribute Decision-Making Approach for Improvisational Emergency Supplier Selection: Partial Ordinal Priority Approach

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Abstract

The frequency of disaster occurrences highlights the importance of scientifically and logically selecting emergency suppliers to ensure the timely distribution of relief provisions to affected regions. The abrupt, unpredictable, and intricate nature of disasters presents formidable challenges to the improvisational emergency supplier selection (IESS). It demands swift decision-making within constrained time frames amidst information scarcity, necessitating coordinating various stakeholders and identifying potential Pareto optimal solutions. Therefore, this study proposes a novel multi-attribute decision-making (MADM) approach, the Partial Ordinal Priority Approach (POPA), tailored to address the challenges inherent in IESS. POPA utilizes easily accessible and stable ranking data, encompassing expert preference information, as model input. This study derives the decision weight optimization model and the adversarial Hasse diagram of the partial-order cumulative transformation set based on ranking preference information. POPA can simultaneously determine the weights of experts, criteria, and alternatives, generating the adversarial Hasse diagram. This diagram streamlines the redundant dominance structure among alternatives and furnishes information on Pareto optimal alternatives, suboptimal alternatives, and alternative clustering details. To validate the effectiveness of POPA, a case study on IESS for the Zhengzhou mega-rainstorm disaster is conducted with sensitivity and comparative analysis. Overall, POPA facilitates swift and stable decision-making while considering potential Pareto optimal solutions amidst time constraints, high information uncertainty, and involvement of multiple stakeholders.

Keywords: Improvisational emergency supplier selection (IESS), Partial Ordinal Priority Approach (POPA), Multi-attribute decision-making (MADM), Partial-order relationship, Adversarial Hasse diagram

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January 29, 2024

1. Introduction

In today's high-risk society, disasters such as earthquakes, floods, terrorist attacks, and pandemics pose severe threats to human life, property, and social stability (Wang et al., 2022). These disasters often exhibit characteristics of suddenness, uncertainty, and complexity, making accurate prediction and control difficult (Song et al., 2024). When disasters strike, they often result in infrastructure damage, transportation disruptions, communication breakdowns, and shortages of emergency supplies, causing significant loss of life and property (Zhang et al., 2022). For instance, during the outbreak of the COVID-19 pandemic in Wuhan, Hubei province, the unique nature and rapid spread of the virus severely impacted the medical system, leading to a shortage of medical resources in the region. The surge of infected patients overwhelmed hospitals, and there was a scarcity of personal protective equipment like medical masks and protective suits, putting frontline medical workers at a high risk of infection. They even had to reuse protective equipment, further increasing the risk of transmission. Two months after the COVID-19 outbreak, the country allocated 40 million medical masks, 5 million sets of protective suits, and 5,000 sets of infrared thermometers to meet the urgent needs of healthcare personnel in Hubei province (People'sDaily, 2020). During disaster response and emergencies, the stability and reliability of the supply chain are crucial to ensure the timely delivery of supplies to affected areas (Liu et al., 2022a). Swift, stable, and sufficient emergency supplies can shorten response times, enhance rescue effectiveness, ensure personnel safety, and reduce disaster losses. Therefore, the scientific and rational selection of emergency suppliers, a typical multi-attribute decision-making (MADM) problem, is essential to emergency decision-making (Zhang et al., 2022; Liao et al., 2020; Mahmoudi and Javed, 2022). However, in the face of extreme disasters, there is often a need for improvisational emergency supplier selection (IESS), when pre-disaster emergency supplier selection often fails to meet demand. Unlike traditional MADM problems, the characteristics of disasters impose new requirements on IESS: decisions need to be made rapidly in a limited time, with high information uncertainty and the involvement of multiple stakeholders (Su et al., 2022; Pamucar et al., 2022; Li et al., 2022a).

Over the past decade, MADM technique has emerged as a crucial instrument for addressing IESS, capable of addressing inherent conflicting objectives, diverse data, and significant uncertainty within IESS. At present, the predominant approach to tackling the problem of IESS relies on MADM methods such as GRA (Zhang et al., 2022), TOPSIS (Afrasiabi et al., 2022; Ge et al., 2020), VIKOR (Zhu and Wang, 2023; Zhang et al., 2023) (Zhu and Wang, 2023; Zhang et al., 2023), TODIM (Liu et al., 2022a; Su et al., 2022; Wang et al., 2023), BWM (Tavakoli et al., 2023; Song et al., 2024), DEMATEL (Gökler and Boran, 2023; Wu and Liao, 2024). Specifically, due to the limited availability and challenging acquisition of objective data in the decision-making process of IESS, current research predominantly relies on subjective decision data from experts. The primary forms include evaluation values (Ge et al., 2020; Sureeyatanapas et al., 2018), semantic values (Li et al., 2022b; Pamucar et al., 2020; Wang et al., 2023), and pairwise comparison values (Wang et al., 2023; Yang et al., 2020). Furthermore, some research extends existing MADM methods using grey system theory (Yang et al., 2020; Zhang et al., 2022), fuzzy set theory (Ge et al., 2020; Qin and Liu, 2019), rough set theory (Rong and Yu, 2024; Jiang et al., 2020; Sun et al., 2020) to address the high uncertainty and substantial ambiguity in the decision-making process of IESS. When aggregating opinions and preferences from multiple stakeholders, most studies employ average-based techniques, such as weighted averages, fuzzy averages, and mean square deviation methods (Liu et al., 2022a; Ning et al., 2022; Zulqarnain et al., 2021). Only a few studies utilize social network analysis to aggregate expert opinions from the perspective of consensus and divergence within the expert group (Liu et al., 2022b).

Through the literature review, four critical limitations in current research have been identified: (1) Most existing MADM methods in IESS only provide a total-order ranking of alternatives, lacking the capability to identify potential Pareto optimal solutions. However, in the practical application of IESS, it is crucial to enhance decision-making transparency and reliability by effectively identifying and prioritizing Pareto optimal alternatives (Grierson, 2008). (2) Most existing MADM methods in IESS overly rely on subjective opinions from experts in the form of evaluative values and pairwise comparison values (Afrasiabi et al., 2022). Nevertheless, stakeholders from different backgrounds demonstrate differences in professional knowledge and the extent of available information. This variability can lead to notable discrepancies in expert judgments, and obtaining these opinions may involve considerable time consumption. (3) Most existing MADM methods in IESS heavily rely on algebraic logic for integrating viewpoints of multiple stakeholders (Ataei et al., 2020). This reliance may lead to neglecting the valuable insights and preferences of experts, resulting in decision outcomes deviating from the actual circumstances. Consequently, it adversely affects the accuracy and effectiveness of the IESS process. (4) Most existing MADM methods in IESS often integrate techniques of data standardization, expert opinion aggregation, and pre-acquisition of weight information based on classical MADM methods. Nevertheless, the above approach unavoidably amplifies the complexity and error probability of MADM models.

To overcome the above limitations in IESS, this study endeavors to introduce a MADM approach (1) utilizing more available and stable decision data as IESS inputs to the MADM model; (2) dispensing with the necessity for integrating data standardization, expert opinion aggregation, or pre-weight acquisition approaches; and (3) expeditiously generating decision outcomes for experts, criteria, and alternatives in MADM, while considering Pareto optimality condition. Therefore, this study proposes the Partial Ordinal Priority Approach (POPA) for solving the IESS problem. Specifically, the proposed approach is based on the Ordinal Priority Approach (OPA) with the ranking data of the criteria, experts, and alternatives as model inputs. Based on the preference information embedded in ranking data, this study derives a decision weight optimization model and partial-order cumulative transformation to generate the most simplified dominance structure of alternatives (adversarial Hasse diagram). The proposed approach can simultaneously determine the weight of experts, criteria, and alternatives and generate dominance structure with information on Pareto optimal alternatives, sub-optimal alternatives, and clustering details. Ultimately, the computed weight outcomes are integrated with the adversarial Hasse diagram to address the IESS while considering the Pareto optimality condition and expert preference information.

The remaining parts of this paper are organized as follows: Section 2 conducts a literature review of the MADM approach in IESS. Section 3 outlines the criteria for IESS. In Section 4, the research method (POPA) is presented. Section 5 employs the IESS process for the Zhengzhou mega-rainstorm disaster as a case study to demonstrate and validate POPA. Lastly, Section 6 presents the conclusions and outlines future directions.

2. Literature Review

When confronted with natural disasters or other emergencies, organizations and governments require swift access to emergency supplies to cater to people's fundamental needs and facilitate disaster relief operations (Zhang et al., 2022). The process of selecting emergency suppliers assumes a pivotal role, as their product quality, service responsiveness, and supply chain reliability directly impact the efficacy of rescue operations and the well-being of those affected (Chen et al., 2020). The selection of emergency suppliers can be conceptualized as a classical MADM problem (Liao et al., 2020; Mahmoudi and Javed, 2022; Mousavi et al., 2020; Shakeel et al., 2020). In current research concerning emergency supplier selection, scholars typically execute it in three steps, as illustrated in Fig.1 (Liu et al., 2022a). The initial step involves gathering decision-making data concerning evaluation criteria, experts, and alternatives, encompassing historical records, statistical data, or expert opinions. The subsequent step entails determining the weights assigned to the criteria and experts. Ultimately, rational MADM methods are utilized to ascertain the comprehensive evaluation value for ranking the alternatives. However, it is noteworthy that the selection of emergency suppliers mainly focuses on the emergency preparedness phase, and the decision-making process is typically unrestricted by severe time pressures (Li et al., 2022b; Liu et al., 2022a). Nevertheless, disaster events often entail complexity and uncertainty, leading to the possibility that pre-selected emergency suppliers may not adequately meet the requirements for disaster response (Pamucar et al., 2022). In such instances, improvisational selection of emergency suppliers becomes essential, exhibiting characteristics distinct from conventional emergency supplier selection. Specifically, IESS must efficiently acquire and aggregate expert opinions, considering preferences information, to achieve stable decision outcomes in situations characterized by intense time pressure, poor quality or difficult accessibility of decision data, and the involvement of multiple stakeholders.

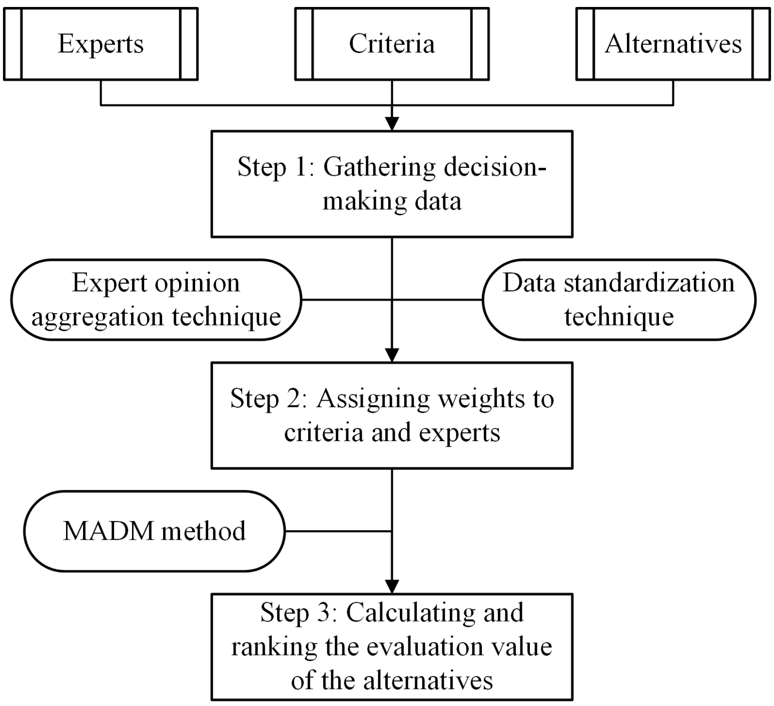

Figure 1: Typical steps of the MADM for IESS

In the present IESS research, widely utilized MADM methods encompass prominent techniques such as GRA (Zhang et al., 2022), TOPSIS (Afrasiabi et al., 2022; Ge et al., 2020), VIKOR (Zhu and Wang, 2023; Zhang et al., 2023), TODIM (Liu et al., 2022a; Su et al., 2022; Wang et al., 2023), BWM (Tavakoli et al., 2023; Song et al., 2024), MARCOS (Rong and Yu, 2024), MACBETH (Pamucar et al., 2020, 2022), DEMATEL (Gökler and Boran, 2023; Wu and Liao, 2024), and others. Considering the challenges associated with acquiring data in emergency situations, only a few studies incorporate objective decision data, while the majority rely on subjective decision data derived from expert opinions due to poor data quality. Subjective decision data takes the form of evaluation values (Ge et al., 2020; Sureeyatanapas et al., 2018; Wang and Cai, 2017), semantic values (Li et al., 2022b; Shakeel et al., 2020; Su et al., 2022; Wang et al., 2023), and pairwise comparison values (Akter et al., 2022; Liu et al., 2022a; Wang et al., 2023; Yang et al., 2020). Given the uncertainty, ambiguity of crises, and limitations in expert experience and available information, the subjective decision data provided by experts also carries a level of uncertainty and ambiguity. Consequently, some studies apply grey system theory (Pamucar et al., 2020; Yang et al., 2020; Zhang et al., 2022), fuzzy theory (Liu et al., 2023; Pamucar et al., 2022; Ge et al., 2020; Qin and Liu, 2019), and rough set theory (Rong and Yu, 2024; Pamucar et al., 2022; Afrasiabi et al., 2022) to enhance subjective decision data, addressing inherent uncertainties and ambiguities. Regarding expert opinion aggregation, most studies mainly rely on algebraic logic-based aggregation techniques, such as weighted averages and geometric means (Ataei et al., 2020; Liu et al., 2022a; Ning et al., 2022; Pamucar et al., 2022; Zulqarnain et al., 2021). Only a few studies explore the application of expert consensus and trust networks in the aggregation of expert opinions in IESS (Liu et al., 2022b). Additionally, to account for risk preferences of decision-makers arising in emergency scenarios, numerous current studies integrate prospect theory (Zhang et al., 2023; Liu et al., 2019), cumulative prospect theory (Liao et al., 2020), and regret theory (Liu et al., 2023) into the IESS decision-making process. For example, in addressing the critical challenge of emergency medical supplier selection during the COVID-19 pandemic, Liu et al. (2022a) employed a comprehensive approach. They integrated expert semantic values transformed into interval Type-2 fuzzy sets (IT2FSs) as inputs and incorporated an extended IT2FSs assessment method along with an original ISM-BWM-Cosine Similarity-Max Deviation Method (IBCSMDM) to account for psychological factors and bidirectional influence relationships. Su et al. (2022) utilized probabilistic linguistic values provided by experts as input data and employed a TODIM approach, augmented with prospect theory, for assessing suppliers, tackling the complexities of MADM in the intricate and dynamic environment. Moreover, Furthermore, in addressing the critical issue of medical supplier selection during the COVID-19 pandemic, Pamucar et al. (2020) proposed a novel decision-making approach called Measuring Attractiveness through a Categorical-Based Evaluation Technique (MACBETH), which incorporates fuzzy rough numbers to effectively manage the inherent high uncertainty associated with the selection process. Zhang et al. (2022) proposed a novel approach, the Spherical Fuzzy Grey Relational Analysis based on Cumulative Prospect Theory (SF-CPT-GRA), which integrates GRA with spherical fuzzy sets and incorporates decision-makers' risk preferences for effective emergency supplier selection.

The essence of the above MADM process lies in projecting the supplier's performance or utility evaluated by experts on various criteria into single comprehensive evaluation criteria for supplier ranking. However, results obtained through the above approach often lack stability and fail to accurately identify potential Pareto-optimal solutions in the IESS

(Kacprzyk et al., 2022). The practice has shown that considering Pareto-optimal solutions in decision analysis can effectively enhance the transparency and stability of decision-making while also providing insights for decision portfolios (Grierson, 2008). In addition, despite the prevailing tendency in most studies to rely on expert opinions, which are more readily available compared to objective decision data, the evaluation values, semantic values, and pairwise comparison values also require the support of substantial professional knowledge and information (Afrasiabi et al., 2022). Experts need to provide specific estimates for alternatives, but this process can be cumbersome and time-consuming, especially when obtaining pairwise comparison values. Moreover, the involvement of multiple stakeholders results in significant variations in decision data. However, the current expert opinion aggregation methods based on algebraic logic often struggle to objectively reflect experts' actual views and preferences in the above context (Ataei et al., 2020). Such disparities may lead to a disconnect between decision outcomes and actual circumstances, thereby impacting the accuracy and effectiveness of supplier selection. Furthermore, the existing IESS methodology, grounded in the traditional MADM process, incorporates a range of methods involving data standardization, expert opinion aggregation, and pre-acquisition of weight information. Undoubtedly, this adds complexity to the model and increases the likelihood of errors. Notably, the dominance relationships among alternatives (i.e., ranking data) are more accessible for experts to establish and more stable and reliable for decision-makers (Wang et al., 2021). This superiority arises as experts only need to assess which one is better without explicitly determining the degree of difference between alternatives. Consequently, employing ranking data as decision data for IESS is a promising approach.

Hence, this research endeavors to present an innovative MADM approach for IESS with uncertainty and intense time pressure. The proposed approach leverages more reliable and easily accessible ranking data as input, eliminating the necessity for data standardization, expert opinion aggregation, and pre-weight acquisition techniques. Additionally, it considers preference information among expert opinions and potential Pareto optimal solutions.

3. Evaluation Criteria of IESS

This section discusses the evaluation criteria for IESS from the perspective of supplier emergency response capability and emergency supply capacity, as outlined in Table 1.

Table 1: Evaluation criteria for IESS

Perspective	Criteria
Supplier emergency response capability	Emergency response speed (C1)
	Emergency delivery reliability (C2)
	Emergency geographic coverage (C3)
	Operation sustainability (C4)
	Collaborative experience and credibility (C5)
Emergency supply capacity	Emergency suppliy availability(C6)
	Emergency supply quality (C7)
	Emergency supply cost-effectiveness (C8)

3.1. Supplier Emergency Response Capability

Disasters entail instability, immediacy, and intricacy, potentially causing disruptions in the supply chain, logistics delays, and inventory deficits. In such situations, the pivotal factor becomes the suppliers' emergency response capability. Broadly, emergency response capability denotes to the suppliers' ability to swiftly respond to and adjust their supply chain, ensuring production and delivery capacity when faced with disasters (Pamucar et al., 2022). Through literature analysis, this study classifies emergency response capability into five core criteria: response speed (C1), delivery reliability (C2), geographic coverage (C3), sustainability (C4), and collaborative experience and credibility (C5). Response speed pertains to the supplier's timeliness in executing emergency response measures (Li et al., 2022a; Zhang et al., 2022). Emergency delivery reliability encompasses the precision and dependability of suppliers in delivering emergency supplies during crises (Afrasiabi et al., 2022; Pamucar et al., 2020). A high level of reliability in emergency supplier deliveries ensures the rapid and accurate distribution of supplies, thereby mitigating losses and impacts arising from disasters. Emergency geographic coverage refers to the extent to which a supplier's emergency supplies can reach affected areas during crises (Mohamadi and Yaghoubi, 2017). Expansive geographic coverage enhances a supplier's ability to promptly deliver emergency supplies and assistance to disaster-stricken regions, thereby minimizing the repercussions of disasters. Sustainability of emergency supplier pertains to their capacity to consistently deliver reliable services and products amidst crises, incorporating sustainable practices across economic, social, and environmental dimensions (Kannan et al., 2020). The collaborative experience and credibility of emergency suppliers refers to the expertise and trustworthiness demonstrated by the supplier in past collaborations when dealing with crises (Li et al., 2022a; Pamucar et al., 2022). Emergency suppliers, drawing from their past cooperative experiences, may be better equipped to handle unforeseen events due to the nature and scale of challenges they may have previously encountered. Moreover, emergency suppliers usually collaborate with multiple organizations, exhibiting excellent coordination and teamwork skills. They are familiar with the communication and collaboration processes with all parties involved, enabling them to cooperate closely with other key stakeholders, thus forming a unified force to respond to unexpected events.

3.2. Emergency Supply Capacity

Emergency supplies capacity primarily focuses on the inherent attributes of emergency supplies that emergency suppliers can provide (Zhang et al., 2022). This capacity can be broadly classified into quality, availability, and cost (Li et al., 2022a; Pamucar et al., 2022). Supply quality encompasses the overall quality and durability, which are critical considerations given the challenging environmental conditions and extensive usage during emergencies (Afrasiabi et al., 2022). High-quality supplies ensure reliability and stability in urgent situations, while low-quality supplies may deteriorate swiftly, hindering or jeopardizing the rescue process and adding further challenges and risks for the affected population. Supply availability pertains to the timely provision and acquisition of supplies during disasters or emergencies (Ge et al., 2020). Reserving an appropriate quantity of supplies ensures swift assistance to affected areas, providing necessary support during critical moments. Supply cost is associated with procurement and usage expenses (Liu et al., 2022a). Exorbitant costs may limit the acquisition and reserves of supplies, impacting the scale and scope of rescue operations. Identifying cost-effective supplies or implementing preemptive storage and procurement measures helps alleviate cost pressures during emergency relief efforts.

4. The Proposed Partial Ordinal Priority Approach

Partial Ordinal Priority Approach (POPA) builds upon the Ordinal Priority Approach (OPA) proposed by Ataei et al. (2020), integrating the partial-order theory and graph theory, thereby representing a partial-order extension of OPA. POPA can be divided into three parts: (1) weight optimization based on ranking preference information, (2) partial-order cumulative transformation, and (3) adversarial Hasse diagram generation. Consequently, POPA yields results regarding the weights of experts, criteria, and alternatives, along with an adversarial Hasse diagram that illustrates the hierarchical dominance structure among alternatives. This section primarily concentrates on elucidating the principle of POPA. Table 2 provides the involved indexes, parameters, sets, variables, and partial-order theory symbols.

Table 2: Notation definition of the proposed approach

Type	Notation	Definition
Index	i	Index of alternatives $(1, \ldots, i, \ldots, m)$
	j	Index of criteria $(1, \ldots, j, \ldots, n)$
	k	Index of experts $(1, \ldots, k, \ldots, p)$
Parameter	re_k	Ranking of the experts k
	rc_{jk}	Ranking of the criteria j under the preference of the expert k
	ra_{ijk}	Ranking of the alternative i for the criteria j under the preference of the expert k
Set	A	Set of alternatives $\forall i \in A$
	C	Set of criteria $\forall j \in C$
	E	Set of expert $\forall k \in E$
Variable	Z	Objective function
	W_{ijk}^{ra}	Weight of the alternative i for the criteria j with the ranking of ra_{ijk} under the preference of the expert k
Partial-order theory symbol	$(A, \preceq_{POCT})$	Partial-order cumulative transformation set (POCTS)
	PR^{POCT}	POCTS in binary matrix form
	$A_{x,POCT}^{+}$	Upper set of the alternative x of POCTS
	$A_{x,POCT}^{-}$	Lower set of the alternative x of POCTS
	$A_{x,POCT}^{\neq}$	Incomparable set of the alternative x of POCTS
	GS^{POCT}	General skeleton matrix of POCTS

4.1. Preliminaries

Definition 1. *Partial-Order Relation*

Let R be a binary relation on a set X, denoted as $R \subseteq X \times X$ (R is a subset of the Cartesian product of X). R is defined as a partial-order relation on set X, denoted as $\preceq$, if it satisfies the following properties:

(1) Self-reversibility: R is self-reversible if xRx for $\forall x \in X$.

(2)Transmissibility: R is transmissible if $xRy, yRz \Rightarrow xRz$ for $\forall x, y, z \in X$.

(3)Antisymmetry: R is antisymmetric if $xRy, yRx \Rightarrow x = y$ for $\forall x, y \in X$.

Definition 2. *Total-Order Relation*

A partial-order relation R on a set X is defined as total-order relation on the set X if it satisfies the strong completeness (i.e., $xRy \vee yRx$ for $\forall x, y \in X$)

The partial-order relation and total-order relation on the alternative set A originating from the utility of the criteria set C are called partial-order set (denoted as $(A, \preceq_C)$) and total-order set (denoted as $(A, \leq_C)$), respectively. Considering evaluations from various experts, $(A, \preceq_{C|E})$ signifies the partial-order set, while the total-order set is expressed in a similar manner. It is worth noting that partial-order relation is a more "flexible" relation compared to total-order relation. This flexibility allows for instances where the compared alternatives are either equivalent or not comparable.

Definition 3. *Lower Set and Upper Set of Partial-Order Set*

Given the partial-order set $(A, \preceq_C)$, for $\forall x \in A$, $A^-_{x,C} = \{y|y \preceq_C x, y \in A\}$ is defined as the lower set of x on the partial-order set $(A, \preceq_C)$, and $A^+_{x,C} = \{y|x \preceq_C y, y \in A\}$ is defined as the upper set of x on the partial-order set $(A, \preceq_C)$.

Property 1. *Given the partial-order set $(A, \preceq_C)$, for $\forall x, y \in A$, there exists: (1) $x \in A^-_{y,C} \Leftrightarrow y \in A^+_{x,C}$; (2) $x \preceq_C y \Leftrightarrow A^-_{x,C} \subseteq A^-_{y,C}$.*

Definition 4. *Order-Preserving Mapping of Partial-Order Set*

Let $(A, \preceq_{C_1})$ and $(B, \preceq_{C_2})$ be the partial-order set, and function $f : A \to B$ is the mapping. If $x \preceq_{C_1} y \Rightarrow f(x) \preceq_{C_2} f(y)$ holds for $\forall x, y, \in A$, then function f is defined as order-preserving mapping of the partial-order set $(A, \preceq_{C_1})$.

Definition 5. *Inclusion Relation in Partial-Order Set*

Let A^-_{x,C_1} and B^-_{x,C_2} be the lower set of x on the partial-order set $(A, \preceq_{C_1})$ and $(B, \preceq_{C_2})$, respectively. If $A^-_{x,C_1} \subseteq B^-_{x,C_2}$ holds for $\forall x \in A$, then it is defined that the partial-order set $(A, \preceq_{C_1})$ is a subset of the partial-order set $(B, \preceq_{C_2})$, denoted as $(A, \preceq_{C_1}) \subseteq (B, \preceq_{C_2})$.

Theorem 1. *Suppose that the partial-order set $(A, \preceq_{C_1}) \subseteq (B, \preceq_{C_2})$ and $A = B$. If $x \preceq_{C_1} y$ holds for $\forall x, y \in A$, then there exists $x \preceq_{C_2} y$.*

PROOF OF THEOREM 1. Since $\forall x, y \in A$, then $\forall x, y \in B$. Given $(A, \preceq_{C_1}) \subseteq (B, \preceq_{C_2})$, by Definition 5, it follows that $A^-_{x,C_1} \subseteq B^-_{x,C_2}, A^-_{y,C_1} \subseteq B^-_{y,C_2}$ for $x, y \in A$. By Property 1, $x \preceq_{C_1} y$ implies that $x \in A^-_y$. And since $A^-_{y,C_1} \subseteq B^-_{y,C_2}$, it follows that $x \in B^-_{y,C_2}$ such that $x \preceq_{C_2} y$. □

Theorem 1 provides an insight that when two partial-order sets, sharing the same alternatives yet differing in criteria, exhibit an inclusive relation, the evaluation outcomes are order-preserving in the context of MADM. In addition, according to Property 1 of the partial-order set, it can be inferred that if two alternatives are comparable, no matter how the weights of experts and criteria change, the partial-order relation between the two alternatives remains unchanged. The above insight will serve as the basis for constructing the partial-order set in POPA that incorporates the information of criteria weight.

4.2. Weight Optimization Based on Ranking Preference Information

In MADM, a pivotal concern is accurately determining the weights assigned to experts, criteria, and alternatives while considering the experts' preference. The weight calculation procedure in POPA is mainly based on the original OPA model with ranking data as input.

The proposed approach formulates a weight optimization model from partial-order theory, facilitating the integration of expert preference information without requiring averaging and data standardization techniques.

When employing POPA, the decision-maker initiates the process by assigning the ranking re_k to each expert. Subsequently, each expert should independently provide the ranking for each criteria rc_{jk},and the ranking for each alternative under each criteria ra_{ijk}. Suppose that A_{ijk}^{ra} is the alternative i with the ranking ra_{ijk} of the criteria j under the evaluation of the expert k. The above ranking provided by the experts independently can reflect the experts' preferences. Then, the partial-order relation among alternatives under each expert and criteria can be presented as Eq.(1).

$$A_{ijk}^{ra=m} \preceq_{C|E} A_{ijk}^{ra=m-1} \preceq_{C|E} \cdots \preceq_{C|E} A_{ijk}^{ra=r+1} \preceq_{C|E} A_{ijk}^{ra=r} \preceq_{C|E} \cdots \preceq_{C|E} A_{ijk}^{ra=1} \quad \forall i,j,k \quad (1)$$

Let W_{ijk}^{ra} be the assigned weight or utility of the alternative i under the evaluation of the expert k with the ranking ra_{ijk} of the criteria j. There exists a corollary $W_{ijk}^{ra=r+1} \leq W_{ijk}^{ra=r}$ such that $A_{ijk}^{ra=r+1} \preceq_{C|E} A_{ijk}^{ra=r}$ holds for $\forall i,j,k$ (i.e., for the same criteria and expert, the weight of alternative with the ranking $ra_{ijk} = r$ is greater than or equal to the weight of the alternative with the ranking $ra_{ijk} = r+1$). Then, the following statement holds:

$$\begin{aligned} & A_{ijk}^{ra=m} \preceq_{C|E} A_{ijk}^{ra=m-1} \preceq_{C|E} \cdots \preceq_{C|E} A_{ijk}^{ra=r+1} \preceq_{C|E} A_{ijk}^{ra=r} \preceq_{C|E} \cdots \preceq_{C|E} A_{ijk}^{ra=1} \triangleq \\ & W_{ijk}^{ra=m} \leq W_{ijk}^{ra=m-1} \leq \cdots \leq W_{ijk}^{ra=r+1} \preceq W_{ijk}^{ra=r} \leq \cdots \leq W_{ijk}^{ra=1} \quad \forall i,j,k \end{aligned} \quad (2)$$

The weight disparities among the consecutive rankings of the alternatives stated in Eq.(2) can be segregated into m autonomous inequalities, as shown in Eq.(3).

$$\begin{aligned} & W_{ijk}^{ra=1} - W_{ijk}^{ra=2} \geq 0 \\ & W_{ijk}^{ra=2} - W_{ijk}^{ra=3} \geq 0 \\ & \dots \\ & W_{ijk}^{ra=r} - W_{ijk}^{ra=r+1} \geq 0 \\ & \dots \\ & W_{ijk}^{ra=m-1} - W_{ijk}^{ra=m} \geq 0 \end{aligned} \quad (3)$$

To evaluate the impact of ranking preference information of experts on the weight of the alternatives, POPA integrates re_k, rc_{jk}, and ra_{ijk} into the assessment of weight disparities of alternatives with consecutive rankings. Thus, this study then multiplies both sides of the inequality (Eq.(3)) by the above ranking parameter, as illustrated in Eq.(4).

$$re_k rc_{jk} ra_{ijk} (W_{ijk}^{ra=r} - W_{ijk}^{ra=r+1}) \geq 0 \quad \forall i,j,k \quad (4)$$

Eq.(1)-Eq.(4) present a logic for analyzing the weight or utility disparities of alternatives with consecutive rankings assigned by experts with preference information. This above process can also be extended to analyze the weight disparities of experts and criteria. It is notable that decision-makers are willing to seek decision weight computations that mirror the expert's preferences and exhibit maximum discrimination. Thus, this study formulates a multi-objective optimization model for decision weight optimization based on the ranking preference information, as shown in Eq.(5). The optimization objectives encompass maximizing the weights of last-ranked alternatives and the weight discrepancies between alternatives with consecutive rankings.

$$\begin{aligned}
&\max\{re_k rc_{jk} ra_{ijk}(W_{ijk}^{ra=r} - W_{ijk}^{ra=r+1}), re_k rc_{jk} ra_{ijk} W_{ijk}^{ra=m}\} \quad \forall i,j,k \\
&s.t. \\
&\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{p} W_{ijk}^{ra} = 1 \\
&W_{ijk}^{ra} \geq 0
\end{aligned} \tag{5}$$

Where the variable W_{ijk}^{ra} is the decision variable of the optimization model and other parameters are consistent with the definition specified in Table 2.

Then, maximum the minimization of the objective function:

$$\begin{aligned}
&\text{maxmin}\{re_k rc_{jk} ra_{ijk}(W_{ijk}^{ra=r} - W_{ijk}^{ra=r+1}), re_k rc_{jk} ra_{ijk} W_{ijk}^{ra=m}\} \quad \forall i,j,k \\
&s.t. \\
&\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{p} W_{ijk}^{ra} = 1 \\
&W_{ijk}^{ra} \geq 0
\end{aligned} \tag{6}$$

The optimization model in max-min form can be further transformed into a linear programming problem by variable substitution:

$$Z = \min\{re_k rc_{jk} ra_{ijk}(W_{ijk}^{ra=r} - W_{ijk}^{ra=r+1}), re_k rc_{jk} ra_{ijk} W_{ijk}^{ra=m}\} \quad \forall i,j,k \tag{7}$$

Substituting Eq.(7) into Eq.(6) yields a linear optimization model for MADM weights based on ranking preferences, as shown in Proposition 1.

Proposition 1. *Decision Weight Optimization Model Based on Ranking Preference Information*

Given the ranking of experts re_k, the ranking of criteria rc_{jk} and alternatives under each criteria ra_{ijk} independently given by the experts, the decision weight optimization model based on ranking preference information of experts is formulated as Eq.(8).

$$\begin{aligned}
&\max Z \\
&s.t. \\
&Z \leq re_k rc_{jk} ra_{ijk}(W_{ijk}^{ra=r} - W_{ijk}^{ra=r+1}) \quad \forall i,j,k \\
&Z \leq re_k rc_{jk} ra_{ijk} W_{ijk}^{ra=m} \quad \forall i,j,k \\
&\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{p} W_{ijk}^{ra} = 1 \\
&W_{ijk}^{ra} \geq 0
\end{aligned} \tag{8}$$

By resolving Eq.(8), the weight of the alternative i with the ranking ra_{ijk} of the criteria j under the evaluation of the expert k can be determined. To ensure the active participation of experts in the decision-making process, it is recommended to set a lower bound of $W_{ijk}^{ra} \geq \eta$, where $\eta = \frac{1}{8 \times m \times n \times p}$.

Eq.(9) provides the formulations for computing the weight of experts, criteria, and alternatives.

$$\begin{aligned} W_i^A &= \sum_{j=1}^{n} \sum_{k=1}^{p} W_{ijk}^{ra} \quad \forall i \\ W_j^C &= \sum_{i=1}^{m} \sum_{k=1}^{p} W_{ijk}^{ra} \quad \forall j \\ W_k^E &= \sum_{i=1}^{m} \sum_{j=1}^{n} W_{ijk}^{ra} \quad \forall k \end{aligned} \tag{9}$$

Through the above derivation, this study obtains an optimization model for determining decision weights while considering experts' ranking preference information. Nevertheless, the computed alternative weights merely project numerous criteria onto a single comprehensive criteria (Yue and Yao, 2023). Consequently, the outcomes lack stability in addressing situations involving Pareto optimal solutions within the alternatives, which is the common problem with the most MADM methods (Cao et al., 2023). To address this limitation, the outcomes of weight optimization based on ranking preference information will undergo a partial-order cumulative transformation in POPA, which incorporates information of criteria weight.

4.3. Partial-Order Cumulative Transformation

The partial-order cumulative transformation of alternative weights in POPA primarily aims to construct a partial-order set incorporating the information of criteria weight, which is more flexible than the partial-order set originating from the strict Pareto optimality condition over each criteria, and more robust than the total-order set based on the single projected comprehensive criteria computed by decision weight optimization model. The core of this process lies in ensuring the newly constructed partial-order set has the property of order-preserving. The first step is to compute alternative weights under the criteria, as depicted in Eq.(10).

$$W_{ij}^{AC} = \sum_{k=1}^{p} W_{ijk}^{ra} \quad \forall i, j \tag{10}$$

Denote $(A, \leq_{SPCC})$ as the total-order set originating from W^A (i.e., single projected comprehensive criteria), and $(A, \preceq_{AC})$ as the partial-order set based on W^{AC} originating from the strict Pareto optimality condition. Given the strict Pareto optimality-based partial-order relation of alternatives, it refers explicitly to the partial-order relation formed by directly examining the unprocessed weights of alternatives on various criteria.

Definition 6. *Partial-Order Cumulative Transformation Set*

Suppose that the criteria in W^{AC} are arranged in descending ranking of the calculated criteria weight. The partial-order cumulative transformation weight can be calculated by Eq.(11). Then, $(A, \preceq_{POCT})$ is defined as the partial-order cumulative transformation set (POCTS) of $(A, \preceq_{AC})$.

$$W_{ij}^{POCT} = \sum_{j=1}^{l} W_{ij}^{AC} \quad l \in [n], and\ \forall i, j \tag{11}$$

The partial-order cumulative transformation weight can be expressed as Eq.(12) in matrix form, where H is the upper triangular matrix. The above procedure represents a linear mapping of cumulative transformations applied to W^{AC}, incorporating weight information.

$$W^{POCT} = W^{AC}H = \begin{bmatrix} W_{11}^{AC} & W_{11}^{AC} + W_{12}^{AC} & \cdots & W_{11}^{AC} + W_{12}^{AC} + \cdots + W_{1n}^{AC} \\ W_{21}^{AC} & W_{21}^{AC} + W_{22}^{AC} & \cdots & W_{21}^{AC} + W_{22}^{AC} + \cdots + W_{2n}^{AC} \\ \vdots & \vdots & \ddots & \vdots \\ W_{m1}^{AC} & W_{m1}^{AC} + W_{m2}^{AC} & \cdots & W_{m1}^{AC} + W_{m2}^{AC} + \cdots + W_{mn}^{AC} \end{bmatrix} \tag{12}$$

Comparing the magnitudes of the row vectors of W^{POCT} yields the POCTS in binary matrix form among the alternatives $PR^{POCT} = [pr_{xy}^{POCT}]_{m\times m}$. Where pr_{xy}^{POCT} can be determined by Eq.(13).

$$pr_{xy}^{POCT} = \begin{cases} 1, & \text{if } W_{xj}^{POCT} \geq W_{yj}^{POCT}, \forall j \\ 0, & \text{otherwise} \end{cases} \quad \forall x, y \in A \tag{13}$$

Subsequently, the order-preserving property of the POCTS is verified by Theorem 2 and Theorem 3.

Theorem 2. *Suppose that $(A, \preceq_{POCT})$ is the partial-order cumulative transformation set of $(A, \preceq_{AC})$, then it follows that $(A, \preceq_{AC}) \subseteq (A, \preceq_{POCT})$.*

Proof of Theorem 2. Let $A_{x,AC}^{-} = \{t | t \preceq_{AC} x, t \in A\}$ and $A_{x,POCT}^{-} = \{t | t \preceq_{POCT} x, t \in A\}$ denote the lower set of $x \in A$ on $(A, \preceq_{AC})$ and $(A, \preceq_{POCT})$, respectively. For $\forall t \in A_{x,AC}^{-}$, there exists $t \preceq_{AC} x \Leftrightarrow W_{tj}^{AC} \leq W_{xj}^{AC}, \forall j$. It follows that $\sum_{j=1}^{l} W_{tj}^{AC} \leq \sum_{j=1}^{l} W_{xj}^{AC}, l \in [n]$, such that $t \in A_{x,POCT}^{-}$, which implies that $A_{x,AC}^{-} \subseteq A_{x,POCT}^{-}$. By Definition 5, $(A, \preceq_{AC}) \subseteq (A, \preceq_{POCT})$ holds. □

Theorem 3. *Suppose that $(A, \preceq_{POCT})$ is the partial-order cumulative transformation set of $(A, \preceq_{AC})$. For $\forall x, y \in A$, when $x \preceq_C y$ exists, $W_{xj}^{POCT} \leq W_{yj}^{POCT}$ holds.*

Proof of Theorem 3. Prove by mathematical induction. Consider W^{AC}, and its elements W_{ij}^{AC} can be decomposed into $W_j^C \cdot V_{ij}^{AC}$, where W_j^C signifies the criteria weight computed in POPA, and V_{ij}^{AC} can be interpreted as an unweighted utility with respect to the criteria j. Given $x \preceq_C y$, it follows that $W_{xj}^{AC} \leq W_{yj}^{AC} \Leftrightarrow V_{xj}^{AC} \leq V_{yj}^{AC}$ such that

$$\begin{aligned} &(W_{y1}^{AC} - W_{x1}^{AC}) + (W_{y2}^{AC} - W_{x2}^{AC}) + \cdots + (W_{yn}^{AC} - W_{xn}^{AC}) \geq 0 \Leftrightarrow \\ &W_1^C(V_{y1}^{AC} - V_{x1}^{AC}) + W_2^C(V_{y2}^{AC} - V_{x2}^{AC}) + \cdots + W_n^C(V_{yn}^{AC} - V_{xn}^{AC}) \geq 0 \end{aligned} \tag{14}$$

When $r = 2$, there exists $W_1^C \geq W_2^C$ and $V_{y1}^{AC} \geq V_{x1}^{AC}$ such that

$$W_1^C(V_{y1}^{AC} - V_{x1}^{AC}) \geq W_2^C(V_{y1}^{AC} - V_{x1}^{AC}) \tag{15}$$

It follows that

$$\begin{aligned} &W_1^C(V_{y1}^{AC} - V_{x1}^{AC}) + W_2^C(V_{y2}^{AC} - V_{x2}^{AC}) \geq \\ &W_2^C(V_{y1}^{AC} - V_{x1}^{AC}) + W_2^C(V_{y2}^{AC} - V_{x2}^{AC}) = \\ &W_2^C(V_{y1}^{AC} - V_{x1}^{AC} + V_{y2}^{AC} - V_{x2}^{AC}) \geq 0 \end{aligned} \tag{16}$$

When $r = l$, there exists $\{W^C_{l-1} \geq W^C_l, \forall j \in l\}$ and $\{V^{AC}_{yj} \geq V^{AC}_{xj}, \forall j \in l\}$ such that

$$\begin{aligned} & W^C_1(V^{AC}_{y1} - V^{AC}_{x1}) + \cdots + W^C_l(V^{AC}_{yl} - V^{AC}_{xl}) \geq \\ & W^C_l(V^{AC}_{y1} - V^{AC}_{x1} + \cdots + V^{AC}_{yl} - V^{AC}_{xl}) \geq 0 \end{aligned} \tag{17}$$

Thus, when $r = n$, there exists:

$$\begin{aligned} & W^C_1(V^{AC}_{y1} - V^{AC}_{x1}) + \cdots + W^C_n(V^{AC}_{yn} - V^{AC}_{xn}) \geq \\ & W^C_n(V^{AC}_{y1} - V^{AC}_{x1} + \cdots + V^{AC}_{yn} - V^{AC}_{xn}) \end{aligned} \tag{18}$$

By the given premise $W^C_1(V^{AC}_{y1} - V^{AC}_{x1}) + W^C_2(V^{AC}_{y2} - V^{AC}_{x2}) + \cdots + W^C_n(V^{AC}_{yn} - V^{AC}_{xn}) \geq 0$, it follows that

$$\begin{aligned} & W^C_1(V^{AC}_{y1} - V^{AC}_{x1}) + \cdots + W^C_n(V^{AC}_{yn} - V^{AC}_{xn}) \geq \\ & W^C_n(V^{AC}_{y1} - V^{AC}_{x1} + \cdots + V^{AC}_{yn} - V^{AC}_{xn}) \geq 0 \end{aligned} \tag{19}$$

Thus, $W^{POCT}_{xj} \leq W^{POCT}_{yj}$ holds. □

Theorem 2 and Theorem 3 prove the order-preserving property of the newly constructed POCTS incorporating information of criteria weight. It is noteworthy that the last column element in W^{POCT} equals to the W^A computed by decision weight optimization model. Then, by Definition 2 of total-order relation, it follows that $(A, \preceq_{AC}) \subseteq (A, \preceq_{POCT}) \subseteq (A, \leq_{SPCC})$. It is further demonstrated by theoretical derivation that the relationship between the constructed POCTS and the original partial-order set based on the strict Pareto optimality condition over each criteria and the total-order set based on the single projected comprehensive criteria. This implies that, theoretically, the partial-order set constructed based on the partial-order cumulative transformation $(A, \preceq_{POCT})$ makes a trade-off between the total-order relation based on comprehensive evaluation weights and the partial-order relation based on the strict Pareto optimality condition, thereby forming a more robust partial-order relation. While the partial-order relation of the strict Pareto optimality condition $(A, \preceq_{AC})$ emphasizes the strict dominance relation between alternatives on each criteria, whereas the total-order relation $(A, \leq_{SPCC})$ is of a stronger property in which each pair of alternatives is comparable. From the perspective of MADM practice, the POCTS signifies that if alternatives face a disadvantage in a relatively important criteria, they may still be deemed viable as long as succeeding criteria can compensate for the deficiency in that specific criteria. This ensures a more stable and dependable outcome for managers when selecting optimal alternatives.

However, based on the transmissibility of the partial-order relation, it is evident that redundant information exists in the generated POCTS binary matrix. It means that the redundant dominance relations can be streamlined according to the transmissibility of the partial-order relations to generate a more concise dominance structure among alternatives. This structure will provide visual support to decision-makers for optimal alternative selection.

4.4. Adversarial Hasse Diagram Generation

In order to streamline the reduction information in the parital-order relation, this study proposes the adversarial Hasse diagram of POPA, drawing inspiration from the Hasse diagram (Wu et al., 2021) and the adversarial interpretative structural modeling (AISM) (Su et al., 2023). Compared to the conventional Hasse diagram, the adversarial Hasse diagram not only identifies the most simplified dominance relations but also extracts hierarchical dominance structures of alternatives based on the non-dominant ascending and descending rules. These dual-direction extraction rules empower decision-makers with a more comprehensive perspective for decision-making. Notably, the dominance structure of alternatives offers intuitive insights into Pareto-optimal alternatives, and clustering hierarchy information of alternatives.

Proposition 2. *Dominant Hierarchy Extraction of Partial-Order Cumulative Transformation Set in Binary Matrix Form*

By Definition 3, the lower set, upper set, and incomparable set of $x \in A$ of the POCTS in binary matrix form can be determined by Eq.(20).

$$\begin{aligned} A^-_{x,POCT} &= \{y|pr^{POCT}_{xy} = 1, y \in A\} \\ A^+_{x,POCT} &= \{y|pr^{POCT}_{yx} = 1, y \in A\} \\ A^{\neq}_{x,POCT} &= A - A^-_{x,POCT} - A^+_{x,POCT} \end{aligned} \tag{20}$$

Regarding dominant hierarchy extraction based on non-dominant ascending rules, if the condition $(A^-_{x,POCT} \cup \{x\}) \cap (A^+_{x,POCT} \cup \{x\}) = (A^-_{x,POCT} \cup \{x\}), x \in A$ is satisfied for $y \in A$, then position y at the top layer. Subsequently, eliminate the rows and columns in PR^{POCT} corresponding to y. Iterate this process, positioning $y \in A$ from the highest to the lowest layer, until all elements in PR^{POCT} are eliminated.

Regarding dominant hierarchy extraction based on non-dominant descending rules, if the condition $(A^-_{x,POCT} \cup \{x\}) \cap (A^+_{x,POCT} \cup \{x\}) = (A^+_{x,POCT} \cup \{x\}), x \in A$ is satisfied for $y \in A$, then position y at the bottom layer. Subsequently, eliminate the rows and columns in PR^{POCT} corresponding to y. Iterate this process, positioning $y \in A$ from the lowest to the highest layer, until all elements in PR^{POCT} are eliminated.

Following Proposition 2, the dominant hierarchy is extracted, and then PR^{POCT} is subjected to edge contraction to eliminate redundant dominance relation information. Following Eq.(21) for edge contraction, the general skeleton matrix (GS) is determined.

$$GS^{POCT} = (PR^{POCT} + I)' - ((PR^{POCT})')^2 - I \tag{21}$$

Where, I is unit matrix, and all operators involved are Boolean operations.

Ultimately, the adversarial Hasse diagram can be derived by substituting the general skeleton matrix into the extracted dominant hierarchy, with its property presented as follows.

Property 2. *Given the adversarial Hasse diagram of the partial-order cumulative transformation set $(A, \preceq_{POCT})$, the following holds:*

(1) The top-layer alternatives, identified through non-dominant ascending and descending rules, represent Pareto optimal alternatives.

(2) The count of dominant hierarchies, determined by non-dominant ascending and descending rules, is uniform, with at least one consistent alternative for each layer.

(3) The dominance relation within the general skeleton matrix maintains transitivity.

(4) The alternatives within the same layer is incomparable.

Through the above derivation, this study formulates POPA and clarifies the motivations and underlying logic behind each part. In general, POPA utilizes more stable and easily obtainable ranking data that mirrors expert preference information, serving as inputs for the model. The decision weight optimization model based on ranking preference information is initially derived to determine the weights of experts, criteria, and alternatives. POPA then proposes a partial-order cumulative transformation to handle the inability to address potential Pareto optimal problems in ranking the alternatives through single projected comprehensive criteria, thus constructing a more robust partial-order relation. Ultimately, by generating the adversarial Hasse diagram, POPA streamlines redundant information within the dominance structure, concurrently presenting Pareto optimal alternatives and the simplified dominance structure with clustering hierarchy information of alternatives. These sequential steps aim to ensure the effectiveness and reliability of the model while addressing potential Pareto optimality scenarios, thereby enhancing the practical guidance and decision support effectiveness of the final decision outcomes.

The steps for implementing POPA are outlined as follows:

Step 1: Determining the experts, criteria, and alternatives.

Step 2: Ranking the experts and acquiring the ranking of criteria and alternatives under each criteria independently provided by each expert.

Step 3: Computing the weight of experts, criteria, and alternatives based on the ranking preference information as expressed in Eq.(8) and Eq.(9).

Step 4: Performing the partial-order cumulative transformation and determining the POCTS in binary matrix form in accordance with Eq.(10), Eq.(11), and Eq.(12).

Step 5: Generating the adversarial Hasse diagram following the process of dominant hierarchy extraction and edge contraction as expressed in Proposition 2 and Eq.(21).

5. Case study and Discussion

5.1. Case Description and Data Collection

This study utilizes the IESS associated with the 7.20 mega-rainstorm disaster in Zhengzhou, China, as a case study to illustrate and validate the proposed POPA. The unprecedented intensity and geographic extent of the torrential rain in Zhengzhou shattered historical precedents, surpassing established flood control and drainage capacities, resulting in widespread waterlogging and inundation (Peng and Zhang, 2022). The exigency of the situation necessitates a substantial influx of medical supplies, sustenance, essential provisions, and other relief materials-outstripping the provisioning capacity of the pre-identified suppliers during the disaster preparedness phase. Confronted with this overwhelming catastrophe, it is necessary to carry out IESS to ensure the seamless advancement of rescue protocols. Considering the tight time constraints, highly uncertain information, and the involvement of multiple stakeholders, the scenario of IESS for the Zhengzhou mega-rainstorm disaster emerges as a representative application scenario for POPA.

Fifteen emergency suppliers, designated as A1 to A15, are at the disposal for selection within the stricken region. These suppliers encompass a spectrum of characteristics, including location, responsiveness, and supply capacity, each exhibiting distinct variations. Certain entities prioritize a stable supply chain and swift responsiveness, albeit at a cost premium beyond the norm. Conversely, other entities boast advantageous geographical placement and efficient traffic connectivity, expediting the provision of flood relief in times of crisis. However, these advantages might be counterbalanced by uncertainties surrounding the quality and durability of the provided rescue materials. These distinguishing characteristics will serve as benchmarks aiding experts in ranking alternatives. Following this, five decision-makers from various departments act as representatives for stakeholders in the selection of emergency suppliers. This group includes representatives from the Zhengzhou Emergency Management Department, the Zhengzhou Civil Affairs Department, and the Zhengzhou Municipal Health Commission. The experts have been prioritized based on their authority in emergency response decision-making, with E5 having the highest rank, followed by E2, E1, E3, and E4 in descending order (E5 >E2 >E1 >E3 >E4). These decision-makers evaluate the essential requirements for specifying emergency suppliers and provide rankings for both sub-criteria and suppliers, as detailed in Table A.1.

5.2. Emergency Supplier Selection Results and Analysis

Figure 2 displays the weights of emergency suppliers, sub-criteria with the computed weight presented in Table B.1. Regarding the expert weight outcomes, Expert E5, possessing the utmost authority, is assigned 0.4380, followed by Expert E2 with 0.2190. Subsequently, Experts E1, E3, and E4 trail behind with weights of 0.1460, 0.1095, and 0.0876, respectively. Notably, the results of expert weights reveal a distinct trend of diminishing marginal effects. In the context of criteria, the most critical factor is the supplier response speed (C1), carrying a weight of 0.2444. Closely followings are supplier collaborative experience and credibility(C5), supplier delivery reliability (C2), and supplier geographic coverage (C3), with weights of 0.2007, 0.1481, and 0.1329, respectively. Conversely, the weighs for the supply availability (C6), the supply quality (C7), the supply cost-effectiveness (C8), and supplier sustainability (C4) are relatively lower, with weights of 0.0931, 0.0694, 0.0602, and 0.0512, respectively. From the criteria weight results of the case study, it is evident that during the improvisational selection of emergency suppliers by critical stakeholders, the primary focus lies on the emergency response capability of suppliers rather than the attributes of the supplies they can provide. Notably, despite the increasing emphasis on the sustainability of humanitarian operations in alignment with UN Sustainable Development Goals, stakeholders in the case study still regard the sustainability of emergency suppliers as a relatively insignificant criteria. Based on the computed weights of alternatives, the top five alternatives are ranked as follows: A8, A3, A7, A5, and A13. Specifically, A8 exhibits the most significant weight, reaching 0.1021, followed by A3, with a weight of 0.0907. In contrast, the weights for A7 and A5 are similar, standing at 0.0829 and 0.0803, respectively. The weight for A13 is 0.072.

However, based on the discussion in the literature review and methodology, the above ranking of alternatives is derived by projecting the evaluation criteria of IESS onto a comprehensive criteria. This outcome raises concerns regarding the stability of decision-making, posing challenges in identifying potential Pareto optimal scenarios. Therefore, for a more comprehensive decision-making insight into the Pareto optimal alternatives and the dominant structure among alternatives, it is imperative to undertake further partial-order cumulative transformation, especially within the context of IESS. Table 3 presents the result of partial-order cumulative transformation.

Through dominant hierarchy extraction and edge contraction operations, the partial-order cumulative transformation set in binary matrix form is streamlined to eliminate redundant information and further generate the adversarial Hasse diagram of emregency suppliers, as depicted in Figure 3. The adversarial Hasse diagram distinctly illustrates the IESS information regarding the dominance structure, Pareto-optimal alternatives, and hierarchical clustering details. The dashed blocks within the adversarial Hasse diagram denote the alternatives with altered hierarchy.

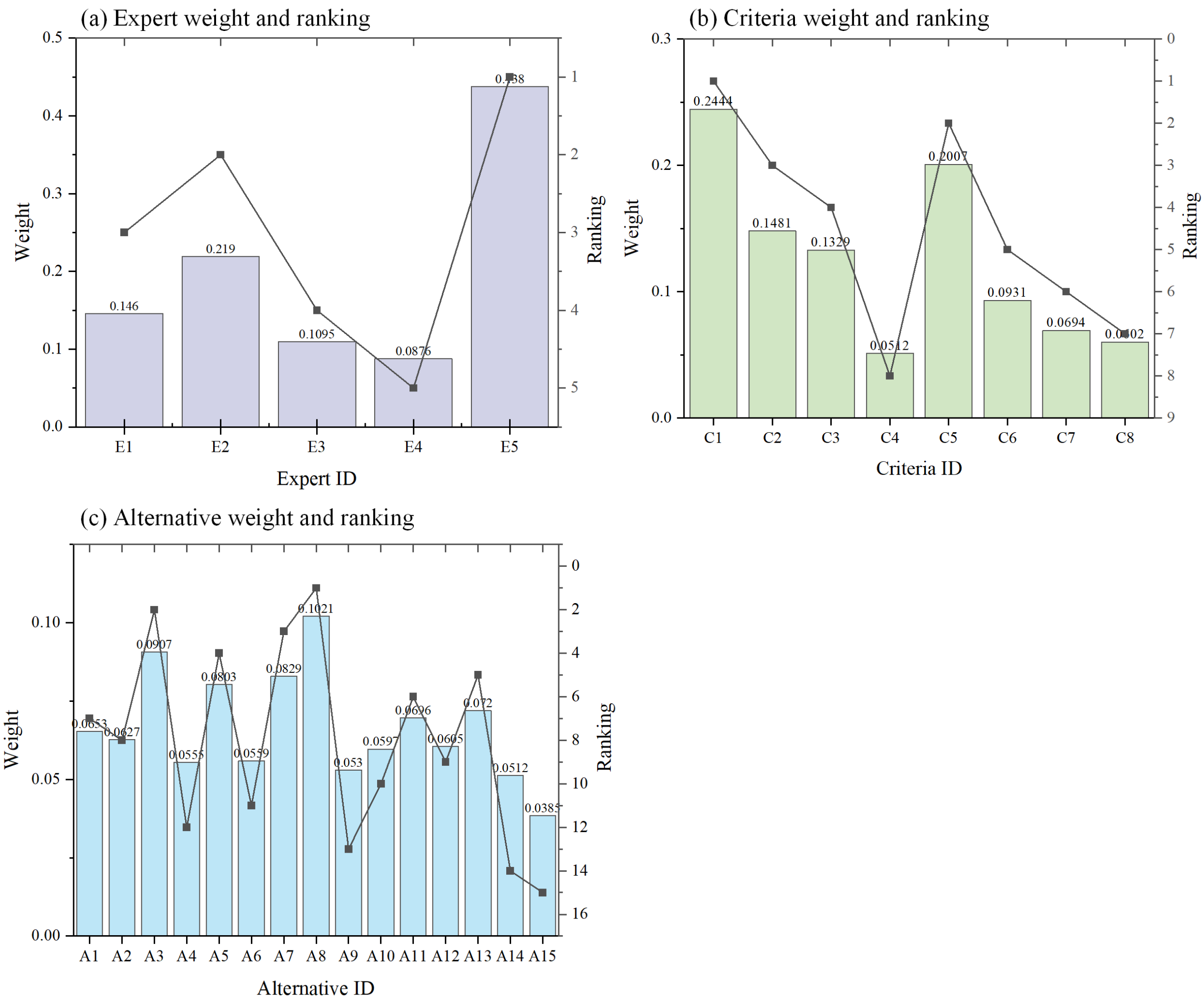

Figure 2: Computed weights of experts, sub-criteria and suppliers of the case IESS

As exemplified in Table 4, the dominant hierarchy of the case IESS unfolds over five layers, elucidating the alternatives with unchanged hierarchy and altered hierarchy inherent within each layer. The structure originating from non-dominant descending rules positions A3 at Layer 1, succeeded by A5 and A8 at Layer 2, and A2, A10, A11, A1, and A13 at Layer 3. Layer 4 encompasses A4, A6, A12, and A14, while Layer 5, the bottom layer, contains A9, A15, and A7. Within the adversarial Hasse diagram based on non-dominant descending rules, the Pareto optimal alternatives for the case IESS emerges as A3, which excels in pivotal dimensions of supplier response speed (C1) and consistently performs exceptionally across other criteria, encompassing supplier delivery reliability (C2) and supply quality (C7). A proximate alternative of note lies in Layer 2, represented by A5 and A8. Notably, under the transitive conditions of the adversarial Haase diagram, A5 serves as a sub-optimal alternative for A3. Regarding structure based on non-dominant ascending rules, the dominance hierarchy of A8 has transitioned from Layer 2 to Layer 1, whereas A7 now occupies Layer 2 rather than Layer 5. This reconfiguration positions A8 and A3 in Layer 1, each with inherent merits and limitations. A comparison between A8 and A3 reveals that, in terms of the supplier collaborative experience and credibility(C5) and the supply cost-effectiveness (C8), A8 takes precedence. Furthermore, A5 and A7 are sub-optimal alter-

Table 3: The results of partial-order cumulative transformation of the case IESS

	$W_{ij_1}^{POCT}$	$W_{ij_2}^{POCT}$	$W_{ij_3}^{POCT}$	$W_{ij_4}^{POCT}$	$W_{ij_5}^{POCT}$	$W_{ij_6}^{POCT}$	$W_{ij_7}^{POCT}$	$W_{ij_8}^{POCT}$
A1	0.0211	0.0301	0.0407	0.0454	0.0531	0.0563	0.0607	0.0653
A2	0.0119	0.0290	0.0351	0.0496	0.0521	0.0556	0.0608	0.0627
A3	0.0342	0.0435	0.0624	0.0722	0.0785	0.0860	0.0887	0.0907
A4	0.0142	0.0173	0.0241	0.0341	0.0418	0.0461	0.0490	0.0555
A5	0.0292	0.0409	0.0539	0.0613	0.0684	0.0735	0.0763	0.0803
A6	0.0121	0.0154	0.0249	0.0330	0.0462	0.0518	0.0538	0.0559
A7	0.0063	0.0337	0.0554	0.0645	0.0721	0.0770	0.0790	0.0829
A8	0.0267	0.0678	0.0719	0.0770	0.0832	0.0862	0.0954	0.1021
A9	0.0080	0.0161	0.0272	0.0363	0.0422	0.0487	0.0516	0.0530
A10	0.0177	0.0318	0.0362	0.0474	0.0509	0.0545	0.0584	0.0597
A11	0.0161	0.0221	0.0302	0.0438	0.0541	0.0602	0.0658	0.0696
A12	0.0112	0.0247	0.0350	0.0449	0.0473	0.0523	0.0560	0.0605
A13	0.0114	0.0338	0.0446	0.0553	0.0620	0.0672	0.0694	0.0720
A14	0.0133	0.0244	0.0321	0.0382	0.0417	0.0444	0.0478	0.0512
A15	0.0111	0.0144	0.0195	0.0230	0.0256	0.0289	0.0358	0.0385

Table 4: Hierarchical clustering information of the case IESS

	Alternatives with unchanged hierarchy	Alternatives with altered hierarchy (ascending rules)	Alternatives with altered hierarchy (descending rules)
Layer 1	A3	—	A8
Layer 2	A5	A8	A7
Layer 3	A1, A2, A10, A11, A13	—	—
Layer 4	A4, A6, A12, A14	—	—
Layer 5	A9, A15	A7	—

natives to A3 and A8. Comparing A5 with A7, the latter excels in the supplier collaborative experience and credibility(C5), supplier delivery reliability (C2), and supplier geographic coverage (C3). Leveraging their respective strengths, A7 demonstrates suitability for large-scale disaster scenarios that require a stable resource supply, whereas A1 is better suited to resource supply scenarios with extreme time pressures.

Therefore, the IESS is not solely determined by the highest weights of alternatives. Projecting multiple criteria onto a comprehensive criteria for the IESS may result in losing valuable decision insights. Instead, this decision-making process necessitates considering the specific capability advantages of each emergency supplier and tailoring the selection to different scenarios. As illustrated by the case study, POPA can provide information on the alternative dominance structure, Pareto-optimal and sub-optimal alternatives, which will aid decision-makers in formulating more transparent and robust decisions. While, this type of information is not possible in some of MADM methods like TOPSIS, VIKOR, and TODIM.

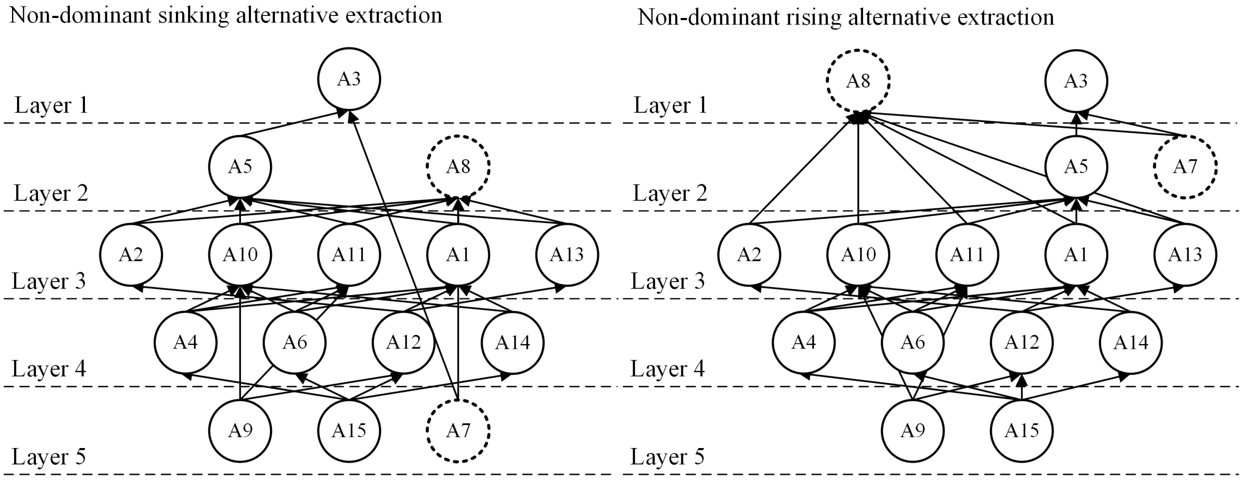

Figure 3: Adversarial Hasse diagram of the case IESS

5.3. Discussion

5.3.1. Validation of POPA Results Based on Sensitivity Analysis

Performing sensitivity analysis on input data or parameters is a vital numerical analysis technique to evaluate the efficacy of MADM methods. The core of POPA involves ranking data with expert preferences, where the rankings of criteria and alternatives under criteria, independently provided by experts, can aptly reflect expert preference information. Analyzing the sensitivity of rankings provided by experts involves perturbing their preference information, thus creating numerous new expert scenarios. Nevertheless, this method lacks a distinct benchmark, impeding thorough analysis. Therefore, this paper conducts a sensitivity analysis on expert rankings supplied by decision-makers to ensure logical coherence. Specifically, this study conducts a complete permutation of rankings for five experts with 120 experiments. The weights assigned to experts, criteria, and alternatives computed by POPA are aggregated. Concurrently, the frequency of instances where each alternative emerges as the Pareto optimal solution is examined within the adversarial Hasse diagram. Figure 4 and Table 5 depict box plots and descriptive statistics of the computed weight outcomes.

In the descriptive statistical analysis of expert weights, the average values obtained by the five experts all demonstrate a consistent feature, standing at 0.2. Expert weights' maximum and minimum values are 0.4380 and 0.0876, respectively. This consistency is also evident across all other indicators. This outcome aligns with intuitive observations from implementing a whole permutation experimental design. Each expert's frequency of occurrence at various ranking positions is equal, resulting in the uniformity of descriptive statistical results among the experts. Regarding the mean values of criterion weights, the most significant is C1, with a weight of 0.2821, followed by C3, with a weight of 0.1717. Subsequently, C5, C2, and C6 closely follow, with weights of 0.1295, 0.1214, and 0.1134, respectively, exhibiting a relatively similar trend. In contrast, the weights of C8 and C4 are the lowest, at 0.0541 and 0.0517, respectively. The Skewness results of the criteria indicate that, except for C1 exhibiting left Skewness, the remaining criteria demonstrate right Skewness. Meanwhile, the Kurtosis results for the criteria reveal that all criteria exhibit negative Kurtosis, implying that the criteria weights concentrate around the mean, with relatively fewer data points in the tails. Notably, the coefficients of variation for both C3 and C5 surpass the designated threshold (0.15), while the coefficients for other criteria

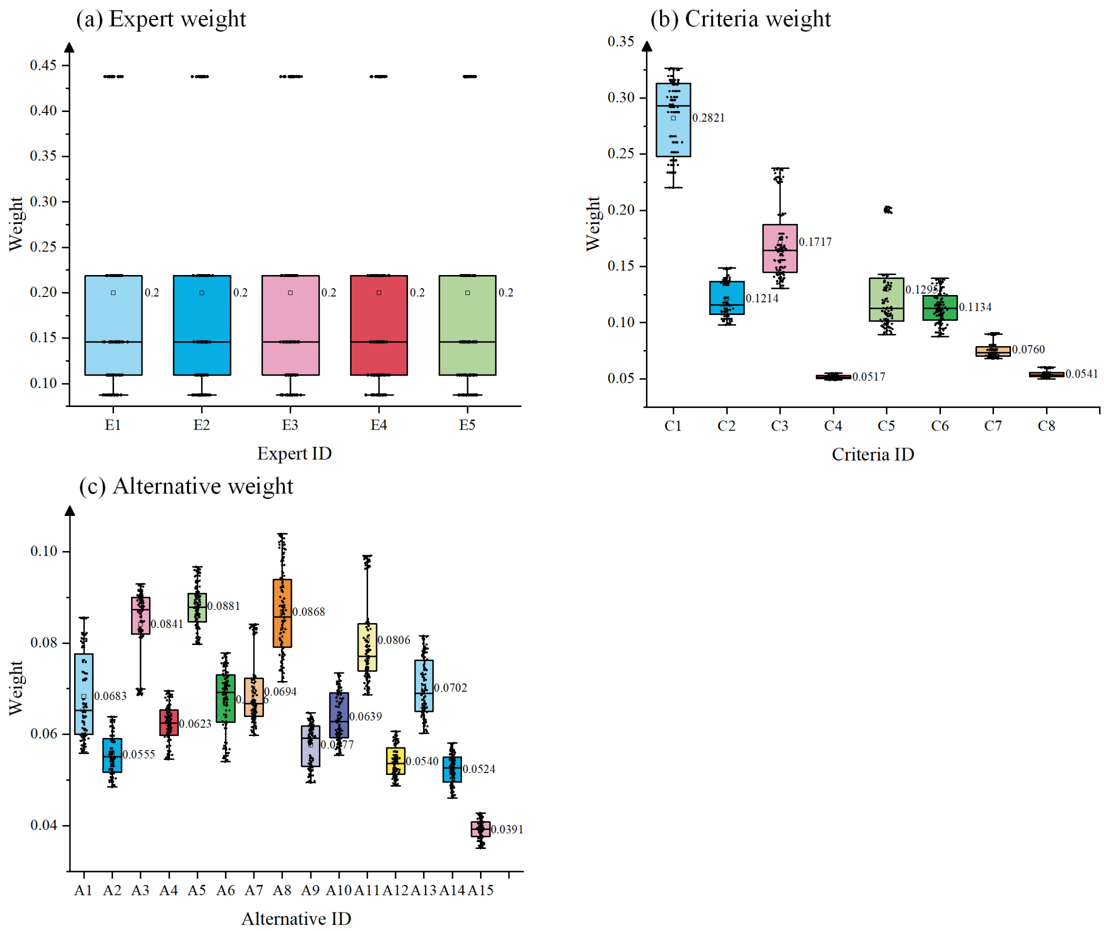

Figure 4: Box plots of the computed weight outcomes

remain below the threshold. Specifically, the coefficient of variation for C5 reaches 0.2956, indicating a pronounced dispersion trend. Furthermore, an examination of Figure 4 reveals a distinct portion of C5 weights reaching 0.2033. This is primarily due to the ranking of C5 provided by expert E5, which is 1, contrasting with rankings of 4, 6, 5, and 7 from other experts. The divergent nature of these evaluations results in abnormal fluctuations in the weight of C5. Upon analyzing the mean weights of alternatives, it is evident that the top four ranked alternatives are A5, A8, A3, and A11, with weights of 0.0881, 0.0868, 0.0841, and 0.0806, respectively. Notably, the maximum weight among the alternatives is associated with A8, reaching 0.1039, while A8 exhibits a relatively high degree of dispersion. Additionally, the coefficients of variation for the weights of the alternatives do not exceed the threshold (0.15). Furthermore, both the standard deviation and variance are smaller than that of the criteria, indicating a more excellent stability of the alternatives compared to the criteria. Furthermore, it is observed that A1, A2, A5, A7, A8, A10, A11, A12, and A13 all display a negative skewness, indicating a right-skewed distribution. This implies that the weight distribution of alternatives with negative Skewness extends more gradually to the right, with the possibility of some relatively large values concentrated overall around the mean. In contrast, the remaining alternatives exhibit characteristics of a left-skewed distribution in their negative Skewness. Through statistical analysis of criteria and alternative weights, it can be deduced that the weight outcomes of POPA are reasonably stable.

Table 5: Descriptive statistics of the computed weight outcomes

	Mean	Max	Min	Standard Deviation	Variance	Skewness	Kurtosis	Coefficient of Variation
E1	0.2000	0.4380	0.0876	0.1276	0.0163	1.1019	-0.3233	0.6380
E2	0.2000	0.4380	0.0876	0.1276	0.0163	1.1019	-0.3233	0.6380
E3	0.2000	0.4380	0.0876	0.1276	0.0163	1.1019	-0.3233	0.6380
E4	0.2000	0.4380	0.0876	0.1276	0.0163	1.1019	-0.3233	0.6380
E5	0.2000	0.4380	0.0876	0.1276	0.0163	1.1019	-0.3233	0.6380
C1	0.2821	0.3263	0.2202	0.0341	0.0012	-0.3725	-1.3224	0.1209
C2	0.1214	0.1488	0.0980	0.0164	0.0003	0.2634	-1.4249	0.1353
C3	0.1717	0.2375	0.1307	0.0331	0.0011	0.8466	-0.6099	0.1927
C4	0.0517	0.0551	0.0490	0.0018	0.0000	0.4307	-0.9298	0.0348
C5	0.1295	0.2033	0.0895	0.0383	0.0015	1.0689	-0.3561	0.2956
C6	0.1135	0.1394	0.0877	0.0144	0.0002	0.0569	-0.8765	0.1267
C7	0.0760	0.0909	0.0680	0.0076	0.0001	1.0011	-0.4367	0.0998
C8	0.0541	0.0605	0.0501	0.0032	0.0000	0.8783	-0.5356	0.0599
A1	0.0683	0.0856	0.0559	0.0093	0.0001	0.4050	-1.2665	0.1362
A2	0.0555	0.0639	0.0485	0.0045	0.0000	0.2496	-1.0751	0.0813
A3	0.0841	0.0929	0.0686	0.0080	0.0001	-1.0532	-0.3691	0.0952
A4	0.0623	0.0696	0.0545	0.0042	0.0000	-0.1595	-0.7932	0.0672
A5	0.0881	0.0967	0.0798	0.0047	0.0000	0.1111	-0.7965	0.0536
A6	0.0676	0.0779	0.0540	0.0070	0.0000	-0.5076	-0.8752	0.1035
A7	0.0694	0.0842	0.0598	0.0076	0.0001	0.9465	-0.4578	0.1089
A8	0.0868	0.1039	0.0715	0.0091	0.0001	0.2538	-0.9885	0.1046
A9	0.0577	0.0647	0.0494	0.0047	0.0000	-0.2852	-1.3624	0.0807
A10	0.0639	0.0734	0.0554	0.0053	0.0000	0.2339	-1.3156	0.0826
A11	0.0806	0.0992	0.0687	0.0095	0.0001	0.9569	-0.4535	0.1178
A12	0.0541	0.0607	0.0488	0.0034	0.0000	0.4575	-1.0154	0.0636
A13	0.0702	0.0816	0.0602	0.0062	0.0000	0.2458	-1.2380	0.0876
A14	0.0524	0.0581	0.0461	0.0033	0.0000	-0.2151	-1.0294	0.0638
A15	0.0391	0.0428	0.0351	0.0021	0.0000	-0.2569	-0.8675	0.0545

Table 6 provides a detailed depiction of the frequencies at which various alternatives emerge as Pareto-optima solutions. It is evident from the observations that A3, A5, A8, A11, and A1 consistently stand out with frequencies exceeding 10 in multiple experiments. Notably, A3, A5, and A8 frequently manifest as Pareto-optimal solutions in experiments exceeding 50 instances. The disclosed information indicates that, under specific circumstances, the above alternatives all have the potential to become Pareto optimal solutions. This finding contrasts significantly with the weight-based total-order ranking results, focusing solely on the alternative with the highest weight. Such differences suggest the importance of considering the potential superiority of alternatives in different contexts during decision-making rather than relying solely on their relative positions in a total-order based on weights. However, the weight-based total-order ranking results exhibit a certain degree of alignment with the alternatives expected to be Pareto optimal. This demonstrates the reliability and rationality of the adversarial Hasse diagram based on partial-order cumulative transformation in identifying Pareto optimality.

Table 6: Frequency of attaining Pareto optimal solutions in adversarial Hasse diagram

	Frequency in non-dominant ascending structure	Frequency in non-dominant descending structure	Discrepancy
A1	13	9	4
A2	0	0	0
A3	99	81	18
A4	0	0	0
A5	82	79	3
A6	0	0	0
A7	0	0	0
A8	51	24	27
A9	0	0	0
A10	3	2	1
A11	33	1	32
A12	0	0	0
A13	6	0	6
A14	6	0	6
A15	0	0	0

5.3.2. Validation of POPA results Based on Comparative Analysis

This study undertakes a comparative analysis within the IESS context of the Zhengzhou mega-rainstorm disaster to validate POPA. Specifically, it contrasts the alternative ranking outcomes of POPA with those derived from conventional MADM methods, encompassing TOPSIS, VIKOR, TODIM, and RSR. Typically, the above MADM methods employ decision matrices with objective values or evaluation scores, necessitating the prior acquisition of criterion weights. Consequently, aligning the ranking outcomes of MADM methods based on the decision matrix with those of POPA facilitates a more efficient demonstration of the merits and validity of POPA. It is worth noting that ELECTRE is not selected due to the additional requirement of setting subjective threshold parameters during its implementation. Regarding the input of the selected MADM methods, the criteria weights produced by POPA are employed for the criteria weight input, with the data presented in Table B.1 serving as the decision matrix. The computed alternative ranking results are shown in Table 7.

Table 7: Alternative ranking results of multiple MADM methods

POPA weight ranking	POPA average dominant hierarchy	TOPSIS	VIKOR	TODIM	RSR
7	3	8	7	6	11
8	3	7	8	8	8
2	1	1	1	2	10
12	4	10	10	11	12
4	2	3	3	3	2
11	4	9	11	10	14
3	3.5	4	4	13	4
1	1.5	2	2	1	1
13	5	13	13	14	9
10	3	11	12	4	7
6	3	5	5	5	6
9	4	12	9	9	5
5	3	6	6	7	3
14	4	14	14	12	13
15	5	15	15	15	15

This paper employs the Spearman correlation coefficient to assess the correlation among the ordinal sequences of alternative ranking results, as shown in Eq.(22).

$$\rho = 1 - \frac{6\sum_{i=1}^{n} d_i^2}{n(n^2-1)} \tag{22}$$

Where n is the number of alternative and d_i^2 is the difference squared between two rankings. As ρ approaches 1, it indicates a pronounced positive correlation between the two rankings; whereas when the ρ approaches -1, it signifies a significant inverse correlation between the two rankings. When ρ approaches 0, it suggests a lack of evident correlation between the two rankings. Table 8 shows the Spearman correlation coefficients of alternative rankings of multiple MADM methods.

The results show that the alternative ranking derived from POPA weights displays a noteworthy positive association with TOPSIS and RSR rankings, with Spearman correlation coefficients of 0.9536 and 0.9750, respectively. In contrast, a moderate correlation is observed with VIKOR and TODIM rankings, featuring corresponding Spearman correlation coefficients of 0.7321 and 0.7500, respectively. The dominant hierarchy of POPA reveals a notable Spearman correlation coefficient of 0.9315 with VIKOR, significantly surpassing correlations with VIKOR from other methods. Furthermore, the dominant hierarchy of POPA shows a robust correlation with TOPSIS and RSR, with corresponding Spearman correlation coefficients of 0.8654 and 0.8452. However, the correlation with TODIM is comparatively lower, at only 0.5806. Notably, the correlation levels of all other methods with TODIM rankings are generally low, ranging between [0.5429, 0.6786]. In summary, the ranking outcomes derived from POPA reveal substantial correlations with other methods, except for some

Table 8: Spearman correlation coefficients of alternative ranking results of multiple MADM methods

	POPA weight ranking	POPA average dominant hierarchy	TOPSIS	VIKOR	TODIM	RSR
POPA weight ranking	1.0000	0.8599	0.9536	0.7321	0.7500	0.9750
POPA average dominant hierarchy	0.8599	1.0000	0.8654	0.9315	0.5806	0.8452
TOPSIS	0.9536	0.8654	1.0000	0.7250	0.6179	0.9714
VIKOR	0.7321	0.9315	0.7250	1.0000	0.5429	0.7214
TODIM	0.7500	0.5806	0.6179	0.5429	1.0000	0.6786
RSR	0.9750	0.8452	0.9714	0.7214	0.6786	1.0000

variations with TODIM. Based on the presented findings, POPA demonstrates reasonable performance compared to other traditional MADM methods, relying on ordinal data that is easily accessible and without the need for prior acquisition of additional weight information.

5.3.3. Advantage Analysis of POPA over Other MADM Methods

Currently, classical MADM methods can be broadly categorized into three main groups: (1) weighting methods, exemplified by entropy-related methods; (2) ranking methods, including TOPSIS, VIKOR, ELECTRE, PROMETHEE, and Hasse diagram; and (3) comprehensive methods, such as AHP, BWM, and OPA. Table 9 presents a comparative analysis between POPA and various other MADM methods across different decision-making characteristics. The table reveals that POPA transforms MADM issues into a mathematical optimization model capable of concurrently determining weights for experts, criteria, and alternatives while identifying Pareto optimal solutions. Furthermore, POPA employs ranking data as model inputs, eliminating the need for pairwise comparison data and decision matrix, where obtaining pairwise comparison data and decision matrix has been proven challenging and time-consuming. Additionally, there is no requirement for standardizing decision data or aggregating expert opinions in POPA.

Table 9: Comparison of POPA with other MADM methods

	AHP/ ANP	Entropy	TOPSIS	TODIM	VIKOR	ELECTRE	PROMETHEE	BWM	Hasse diagram	OPA	POPA
Weighting of criteria	✓	✓	×	×	×	×	×	✓	×	✓	✓
Weighting of experts	×	✓	×	×	×	×	×	×	×	✓	✓
Weighting of alternatives	✓	×	✓	✓	✓	✓	✓	✓	×	✓	✓
Utilizing decision matrix	×	✓	✓	✓	✓	✓	✓	×	✓	×	×
Utilizing pairwise comparison data	✓	×	×	×	×	×	✓	✓	×	×	×
Utilizing ranking data	×	×	×	×	×	×	×	×	×	✓	✓
No translation of qualitative into quantitative variables required	×	×	×	×	×	×	×	×	×	✓	✓
No data standardization required	✓	×	×	×	×	×	✓	✓	×	✓	✓
No expert opinion aggregation required	×	×	×	×	×	×	×	✓	×	✓	✓
No effect of positive and negative ideal solutions on results	✓	✓	×	×	×	×	×	✓	✓	✓	✓
Formulating the problem as a mathematical optimization model	×	×	×	×	×	×	×	✓	×	✓	✓
Ability to undertake group decision-making	✓	✓	✓	✓	✓	✓	✓	✓	×	✓	✓
Ability to address Pareto optimal solutions	×	×	×	×	×	×	×	×	✓	×	✓

6. Conclusion

In the contemporary high-risk society, frequent extreme climate disasters, terrorist attacks, and major infectious diseases introduce notable abruptness and uncertainty, presenting substantial threats to human welfare, economic stability, and the security of individuals' lives and assets. In such scenarios, the rapid surge in demand for emergency supplies often exceeds the existing supply from emergency suppliers. Decision-makers must judiciously and promptly navigate this uncertain environment to make improvisational selections of suitable emergency suppliers. Presently, the MADM within IESS frequently encounters a series of formidable challenges, manifested as follows: (1) insufficient, imprecise, and time-consuming acquisition of decision data; (2) the necessity to coordinate and integrate decision opinions and preferences from multiple stakeholders; (3) the identification of potential Pareto optimal solutions during the decision-making process.

Therefore, this study presents POPA for addressing the challenges that IESS encounters. Specifically, POPA builds upon the OPA with the more accessible and reliable rankings of experts, criteria, and alternatives as the model inputs. The study develops a decision weight optimization model capable of simultaneously obtaining the weights of experts, criteria, and alternatives, considering expert ranking preferences. To enhance decision stability and effectively identify potential Pareto optimal solutions, this study derives the partial-order cumulative transformation of decision weights and the corresponding partial-order cumulative transformation set of alternatives. Subsequently, an adversarial Hasse diagram of the partial-order cumulative transformation set is introduced. This not only streamlines the redundant dominance structure among alternatives but also furnishes information on Pareto optimal alternatives, suboptimal alternatives, and alternative clustering details. Utilizing the IESS problem during the Zhengzhou heavy rain disaster as a case study, this paper presents an illustrative demonstration of POPA and further validates its efficacy through sensitivity and comparative analysis.

The fundamental contribution of this study lies in the introduction of POPA to address the IESS. Methodologically, this study incorporates the partial-order theory into OPA for the first time, achieving a partial-order extension of OPA. Specifically, POPA utilizes easily accessible and stable ranking data from experts as input, making it more suitable for decisions that are characterized by a lack of sufficient and accurate data. POPA facilitates the simultaneous determination of weights for experts, criteria, and alternatives, considering expert preference information without needing data standardization, expert opinion aggregation, or pre-acquisition of criteria weights. Additionally, the partial-order cumulative transformation and adversarial Hasse diagram generation of POPA, supported by theoretical derivation, can effectively identify potential Pareto optimal solutions and facilitate more robust decision-making. Practically, this study utilizes POPA by integrating representative criteria system to address IESS, offering insights and guidance for authentic decision-making processes. Furthermore, POPA is not limited to IESS but is also applicable to any decision scenario exhibiting similar characteristics.

Finally, it is essential to emphasize that the conclusions and findings of this study are based on limited case scenarios. Consequently, there is a need for further exploration of practical applications of POPA to validate its effectiveness. Notably, the implementation of IESS based on POPA relies on the assumption of criteria independence. Therefore, future studies could refine POPA, considering the interplay of criteria for a more accurate reflection of real-world situations. Lastly, this study does not encompass the impact of decision-maker's risk preferences on decision outcomes. Accordingly, future research is expected to incorporate the risk preferences of decision-makers into the model extension of POPA.

Author Contributions

Renlong Wang: Writing - original draft, Conceptualization, Methodology, Validation, Software. **Rui Shen**: Data curation, Validation, Visualization, Formal analysis. **Shutian Cui**: Validation, Visualization, Formal analysis. **Xueyan Shao**: Writing – review & editing, Validation, Investigation. **Hong Chi**: Supervision, Investigation. **Mingang Gao** : Supervision, Writing – review & editing, Funding acquisition, Investigation.

Funding

This research was funded by the National Natural Science Foundation of China (Grant number: 72134004).

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data Availability

Data will be made available on request.

Appendix A.

Table A.1: Ranking of criteria and alternatives under criteria provided by experts of the case IESS

Expert ID	Supplier ID	SC1	SC2	SC3	SC4	SC5	SC6	SC7	SC8
E1	—	1	5	2	7	4	3	6	8
E2	—	1	2	4	8	6	3	5	7
E3	—	1	4	3	8	5	2	6	7
E4	—	2	5	1	6	7	4	3	8
E5	—	3	2	4	7	1	8	6	5
	A1	1	14	10	8	3	6	8	15
	A2	9	10	6	10	15	11	11	5
	A3	2	15	5	15	6	15	10	1
	A4	10	7	4	11	9	5	3	9
	A5	3	4	15	1	4	7	7	13
E1	A6	4	13	3	12	7	8	1	6
	A7	8	3	14	2	5	3	14	8
	A8	7	9	12	3	10	12	4	4
	A9	11	2	11	5	1	2	9	10
	A10	6	12	1	14	11	13	2	3
	A11	12	1	13	4	2	1	12	11

Table A.1: Ranking of criteria and alternatives under criteria provided by experts of the case IESS

Expert ID	Supplier ID	SC1	SC2	SC3	SC4	SC5	SC6	SC7	SC8
	A12	14	6	2	13	14	14	13	2
	A13	5	11	7	9	8	4	15	12
	A14	13	8	8	6	12	9	5	7
	A15	15	5	9	7	13	10	6	14
	A1	13	1	4	11	3	1	15	10
	A2	4	15	11	3	7	13	10	3
	A3	1	14	10	9	12	14	9	6
	A4	7	6	7	4	6	5	5	11
	A5	2	9	12	5	5	8	11	7
	A6	11	4	6	6	11	2	3	12
	A7	15	2	3	12	10	9	4	15
E2	A8	3	7	14	2	1	4	12	4
	A9	14	3	2	15	9	6	1	14
	A10	6	13	13	7	14	11	14	2
	A11	12	5	1	14	15	3	2	13
	A12	5	12	15	1	2	12	13	8
	A13	10	8	5	13	8	7	8	5
	A14	8	10	9	8	4	10	6	9
	A15	9	11	8	10	13	15	7	1
	A1	7	9	14	1	7	13	15	6
	A2	15	6	4	11	12	12	2	5
	A3	2	14	15	2	2	3	10	12
	A4	14	2	1	15	4	2	14	14
	A5	1	13	13	7	3	14	9	1
	A6	13	5	3	12	5	1	1	15
	A7	6	10	12	6	11	10	12	4
E3	A8	3	15	11	3	1	4	11	13
	A9	12	1	2	14	10	9	3	7
	A10	5	11	10	8	13	11	13	3
	A11	10	3	5	4	9	5	4	10
	A12	8	7	8	13	15	15	8	2
	A13	11	4	6	5	6	6	5	8
	A14	4	12	9	9	14	8	7	11
	A15	9	8	7	10	8	7	6	9
	A1	11	6	8	14	15	14	5	4
	A2	6	10	7	13	9	6	4	10
	A3	15	5	12	8	4	7	10	7
	A4	7	9	6	12	14	5	3	11
E4	A5	10	7	3	5	3	13	2	3
	A6	3	4	13	4	13	1	15	15
	A7	12	1	9	9	2	2	9	14
	A8	9	11	4	15	8	8	6	6
	A9	5	8	11	6	10	12	11	5

Table A.1: Ranking of criteria and alternatives under criteria provided by experts of the case IESS

Expert ID	Supplier ID	SC1	SC2	SC3	SC4	SC5	SC6	SC7	SC8
	A10	2	14	5	7	6	9	7	8
	A11	13	3	1	10	5	3	1	12
	A12	8	12	10	1	11	10	14	2
	A13	14	2	2	11	1	4	8	13
	A14	1	13	15	2	12	15	13	1
	A15	4	15	14	3	7	11	12	9
	A1	4	14	15	3	10	15	6	4
	A2	14	7	1	14	4	10	11	5
	A3	10	1	2	12	9	2	1	15
	A4	3	12	11	1	15	14	12	6
	A5	13	3	6	10	8	1	5	14
	A6	9	8	10	11	14	4	15	9
	A7	15	2	3	5	2	3	4	10
E5	A8	2	13	12	2	1	9	10	1
	A9	6	11	7	13	11	13	7	7
	A10	8	9	8	15	5	5	8	13
	A11	1	15	13	4	12	12	13	2
	A12	11	4	4	9	6	6	2	12
	A13	7	5	9	6	3	7	3	11
	A14	12	6	5	7	7	8	14	8
	A15	5	10	14	8	13	11	9	3

Appendix B.

Table B.1: Computed weight outcomes of the case IESS

Expert ID	Supplier ID	C1	C2	C3	C4	C5	C6	C7	C8
	A1	0.0119	0.0001	0.0009	0.0004	0.0016	0.0012	0.0004	0.0000
	A2	0.0021	0.0004	0.0019	0.0003	0.0001	0.0005	0.0002	0.0006
	A3	0.0083	0.0000	0.0022	0.0000	0.0009	0.0001	0.0003	0.0015
	A4	0.0018	0.0006	0.0027	0.0002	0.0005	0.0015	0.0011	0.0003
	A5	0.0065	0.0011	0.0001	0.0017	0.0013	0.0010	0.0005	0.0001
	A6	0.0053	0.0002	0.0033	0.0002	0.0008	0.0009	0.0020	0.0005
	A7	0.0026	0.0013	0.0002	0.0012	0.0011	0.0022	0.0001	0.0003
E1	A8	0.0031	0.0004	0.0005	0.0009	0.0004	0.0004	0.0009	0.0007
	A9	0.0014	0.0017	0.0007	0.0006	0.0030	0.0028	0.0004	0.0002
	A10	0.0037	0.0002	0.0059	0.0001	0.0003	0.0003	0.0014	0.0008
	A11	0.0011	0.0024	0.0004	0.0008	0.0021	0.0040	0.0002	0.0002
	A12	0.0005	0.0007	0.0042	0.0001	0.0001	0.0002	0.0001	0.0010
	A13	0.0044	0.0003	0.0016	0.0003	0.0006	0.0018	0.0000	0.0001
	A14	0.0008	0.0005	0.0013	0.0005	0.0003	0.0007	0.0007	0.0004
	A15	0.0002	0.0009	0.0011	0.0004	0.0002	0.0006	0.0006	0.0001

Table B.1: Computed weight outcomes of the case IESS

Expert ID	Supplier ID	C1	C2	C3	C4	C5	C6	C7	C8
	A1	0.0012	0.0089	0.0020	0.0003	0.0016	0.0059	0.0001	0.0004
	A2	0.0080	0.0002	0.0005	0.0012	0.0008	0.0004	0.0005	0.0014
	A3	0.0178	0.0004	0.0007	0.0004	0.0003	0.0002	0.0006	0.0008
	A4	0.0047	0.0028	0.0012	0.0010	0.0009	0.0022	0.0013	0.0003
	A5	0.0125	0.0016	0.0004	0.0008	0.0011	0.0013	0.0004	0.0007
	A6	0.0021	0.0040	0.0014	0.0007	0.0003	0.0042	0.0020	0.0002
	A7	0.0004	0.0062	0.0024	0.0002	0.0004	0.0011	0.0016	0.0001
E2	A8	0.0098	0.0023	0.0002	0.0016	0.0030	0.0027	0.0003	0.0011
	A9	0.0007	0.0049	0.0031	0.0000	0.0005	0.0019	0.0036	0.0001
	A10	0.0056	0.0006	0.0003	0.0006	0.0001	0.0007	0.0001	0.0018
	A11	0.0016	0.0033	0.0045	0.0001	0.0001	0.0033	0.0025	0.0002
	A12	0.0066	0.0008	0.0001	0.0022	0.0021	0.0005	0.0002	0.0006
	A13	0.0026	0.0019	0.0017	0.0001	0.0006	0.0016	0.0008	0.0009
	A14	0.0039	0.0013	0.0008	0.0005	0.0013	0.0009	0.0011	0.0005
	A15	0.0032	0.0010	0.0010	0.0003	0.0002	0.0001	0.0009	0.0025
	A1	0.0023	0.0004	0.0001	0.0011	0.0005	0.0003	0.0000	0.0004
	A2	0.0002	0.0007	0.0013	0.0001	0.0002	0.0004	0.0010	0.0005
	A3	0.0062	0.0001	0.0001	0.0008	0.0012	0.0024	0.0002	0.0001
	A4	0.0004	0.0016	0.0030	0.0000	0.0008	0.0031	0.0001	0.0001
E3	A5	0.0089	0.0001	0.0002	0.0003	0.0010	0.0002	0.0003	0.0013
	A6	0.0006	0.0008	0.0016	0.0001	0.0007	0.0045	0.0015	0.0000
	A7	0.0028	0.0003	0.0003	0.0003	0.0002	0.0007	0.0001	0.0006
	A8	0.0049	0.0000	0.0003	0.0006	0.0018	0.0020	0.0002	0.0001
	A9	0.0008	0.0022	0.0021	0.0000	0.0003	0.0008	0.0008	0.0003
	A10	0.0033	0.0003	0.0004	0.0002	0.0001	0.0005	0.0001	0.0007
	A11	0.0013	0.0012	0.0011	0.0005	0.0003	0.0017	0.0007	0.0002
	A12	0.0019	0.0006	0.0006	0.0001	0.0000	0.0001	0.0003	0.0009
	A13	0.0010	0.0010	0.0009	0.0004	0.0006	0.0014	0.0006	0.0003
	A14	0.0040	0.0002	0.0005	0.0002	0.0001	0.0010	0.0004	0.0001
	A15	0.0016	0.0005	0.0008	0.0002	0.0004	0.0012	0.0005	0.0002
	A1	0.0004	0.0004	0.0016	0.0000	0.0000	0.0001	0.0009	0.0004
	A2	0.0011	0.0002	0.0019	0.0001	0.0002	0.0006	0.0011	0.0001
	A3	0.0001	0.0005	0.0006	0.0003	0.0005	0.0005	0.0004	0.0002
	A4	0.0009	0.0003	0.0022	0.0001	0.0000	0.0007	0.0013	0.0001
	A5	0.0005	0.0004	0.0039	0.0004	0.0006	0.0001	0.0017	0.0005
	A6	0.0020	0.0006	0.0005	0.0005	0.0001	0.0018	0.0000	0.0000
	A7	0.0003	0.0014	0.0013	0.0002	0.0007	0.0012	0.0004	0.0000
E4	A8	0.0006	0.0002	0.0032	0.0000	0.0002	0.0004	0.0007	0.0003
	A9	0.0013	0.0003	0.0008	0.0004	0.0002	0.0002	0.0003	0.0003
	A10	0.0025	0.0001	0.0027	0.0003	0.0003	0.0003	0.0006	0.0002
	A11	0.0002	0.0008	0.0071	0.0002	0.0004	0.0010	0.0024	0.0001
	A12	0.0008	0.0001	0.0011	0.0012	0.0001	0.0003	0.0001	0.0006
	A13	0.0001	0.0010	0.0050	0.0001	0.0010	0.0008	0.0005	0.0001

Table B.1: Computed weight outcomes of the case IESS

Expert ID	Supplier ID	C1	C2	C3	C4	C5	C6	C7	C8
	A14	0.0036	0.0001	0.0001	0.0008	0.0001	0.0000	0.0002	0.0009
	A15	0.0016	0.0000	0.0003	0.0007	0.0003	0.0002	0.0002	0.0002
	A1	0.0053	0.0007	0.0002	0.0028	0.0053	0.0001	0.0019	0.0032
	A2	0.0005	0.0047	0.0089	0.0002	0.0160	0.0007	0.0007	0.0027
	A3	0.0018	0.0178	0.0062	0.0005	0.0064	0.0031	0.0059	0.0001
	A4	0.0065	0.0016	0.0010	0.0051	0.0007	0.0002	0.0005	0.0022
	A5	0.0008	0.0098	0.0028	0.0008	0.0078	0.0045	0.0022	0.0003
	A6	0.0021	0.0039	0.0013	0.0006	0.0015	0.0020	0.0001	0.0013
	A7	0.0002	0.0125	0.0049	0.0019	0.0249	0.0024	0.0027	0.0011
E5	A8	0.0083	0.0012	0.0008	0.0036	0.0356	0.0008	0.0009	0.0071
	A9	0.0037	0.0021	0.0023	0.0003	0.0042	0.0003	0.0016	0.0019
	A10	0.0026	0.0032	0.0019	0.0001	0.0133	0.0017	0.0013	0.0005
	A11	0.0119	0.0004	0.0006	0.0023	0.0032	0.0004	0.0004	0.0050
	A12	0.0014	0.0080	0.0040	0.0009	0.0111	0.0014	0.0042	0.0006
	A13	0.0031	0.0066	0.0016	0.0016	0.0195	0.0012	0.0033	0.0008
	A14	0.0011	0.0056	0.0033	0.0013	0.0093	0.0010	0.0002	0.0016
	A15	0.0044	0.0026	0.0004	0.0011	0.0023	0.0005	0.0011	0.0039

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